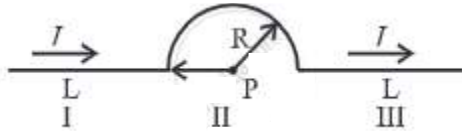


Chapter 10: Magnetic Fields due to Electric Current

EXERCISES [PAGES 248 - 250]

Exercises | Q 1.1 | Page 248

A conductor has 3 segments; two straight and of length L each and a semicircular with radius R . It carries a current I . What is the magnetic field B at point P ?



$$\frac{\mu_0 I}{4\pi R}$$
$$\frac{\mu_0 I}{4\pi R^2}$$
$$\frac{\mu_0 I}{4R}$$
$$\frac{\mu_0 I}{4\pi}$$

SOLUTION

$$\frac{\mu_0 I}{4R}$$

Explanation:

Magnetic field due to the straight portion of the wire = 0

Magnetic field due to circular portion of the wire:

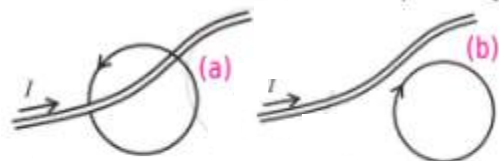
$$= \frac{1}{2} \left(\frac{\mu_0 I}{2} R \right)$$
$$= \frac{\mu_0 I}{4R}$$

Exercises | Q 1.2 | Page 248

Choose the correct option:

Figures (a) and (b) show two Amperean loops associated with the conductors carrying current I in the sense shown.

The $\oint \vec{B} \cdot d\vec{l}$ in the cases (a) and (b) are, respectively.



$-\mu_0 I, 0$

$\mu_0 I, 0$

$0, \mu_0 I$

$0, -\mu_0 I$

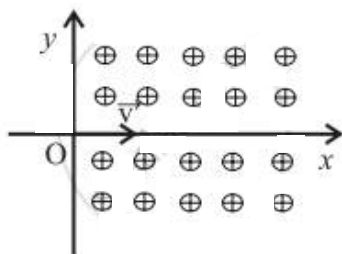
SOLUTION

$-\mu_0 I, 0$

Exercises | Q 1.3 | Page 248

Choose the correct option.

A proton enters a uniform magnetic field B , directed into the plane of the page, perpendicularly along the positive x-axis with a velocity v , as shown. Then, it will follow the following path. [The magnetic field is directed into the paper].



1. It will continue to move along positive x-axis.
2. **It will move along a curved path, bending towards positive y-axis**
3. It will move along a curved path, bending towards negative y-axis
4. It will move along a sinusoidal path along the positive x-axis

SOLUTION

It will move along a curved path, bending towards positive y-axis

Exercises | Q 1.4 | Page 249

Choose the correct option.

A conducting thick copper rod of length 1 m carries a current of 15 A and is located on the Earth's equator. There the magnetic flux lines of Earth's magnetic field are horizontal, with the field of 1.3×10^{-4} T, south to north. The magnitude and direction of the force on the rod, when it is oriented so that current flows from west to east are

1. 14×10^{-4} N, downward
2. 20×10^{-4} N, downward
3. 14×10^{-4} N, upward
4. **20×10^{-4} N, upward**

SOLUTION

20×10^{-4} N, upward

Exercises | Q 1.5 | Page 249

A charged particle is in motion having initial velocity v when it enters into a region of uniform magnetic field perpendicular to v . Because of the magnetic force the kinetic energy of the particle will

1. **remain unchanged**
2. get reduced
3. increase
4. be reduced to zero

SOLUTION

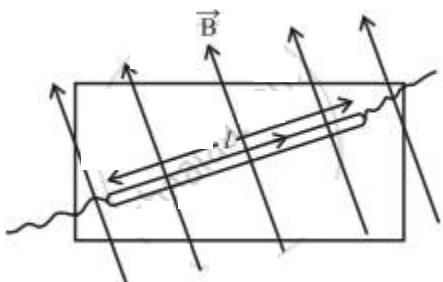
remain unchanged

Explanation:

The magnetic force acts perpendicular to the velocity of the particle. This causes circular motion. In the magnetic field, the speed and kinetic energy of the particle remains constant, but the direction is altered at each instant by the perpendicular magnetic force. Hence the kinetic energy remains constant.

Exercises | Q 2 | Page 249

A piece of straight wire has mass 20 g and length 1 m. It is to be levitated using a current of 1 A flowing through it and a perpendicular magnetic field B in a horizontal direction. What must be the magnitude B of the magnetic field?



SOLUTION

Data: $m = 20 \text{ g} = 2 \times 10^{-2} \text{ kg}$, $l = 1 \text{ m}$, $I = 1 \text{ A}$, $g = 9.8 \text{ m/s}^2$

To balance the wire, the upward magnetic force must be equal in magnitude to the downward force due to gravity.

$$\therefore F_m = IlB = mg$$

Therefore, the magnitude of the magnetic field,

$$B = \frac{mg}{Il} = \frac{(2 \times 10^{-2})(9.8)}{(1)(1)} = 0.196 \text{ T}$$

Exercises | Q 3 | Page 249

Calculate the value of the magnetic field at a distance of 2 cm from a very long straight wire carrying a current of 5 A (Given: $\mu_0 = 4\pi \times 10^{-7} \text{ Wb/Am}$).

SOLUTION

Data: $I = 5 \text{ A}$, $a = 0.02 \text{ m}$, $\frac{\mu_0}{4\pi} = 10^{-7} \text{ T m/A}$

The magnetic induction,

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{4\pi} \frac{2I}{a} = 10^{-7} \times \frac{2(5)}{2 \times 10^{-2}} = 5 \times 10^{-5} \text{ T}$$

Exercises | Q 4 | Page 249

An electron is moving with a speed of $3 \times 10^{-7} \text{ m/s}$ in a magnetic field of $6 \times 10^{-4} \text{ T}$ perpendicular to its path. What will be the radius of the path? What will be frequency and the energy in keV?

[**Given:** mass of electron = $9 \times 10^{-31} \text{ kg}$, charge $e = 1.6 \times 10^{-19} \text{ C}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$]

SOLUTION

Data: $v = 3 \times 10^7$ m/s, $B = 6 \times 10^{-4}$ T,

$m_e = 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C,

1 eV = 1.6×10^{-19} J

The radius of the circular path,

$$\begin{aligned} r &= \frac{m_e v}{|e|B} \\ &= \frac{(9 \times 10^{-31})(3 \times 10^7)}{(1.6 \times 10^{-19})(6 \times 10^{-4})} = \frac{2.7}{9.6} = 0.2812 \text{ m} \end{aligned}$$

The frequency of revolution,

$$\begin{aligned} f &= \frac{|e|B}{2\pi m_e} \\ &= \frac{(1.6 \times 10^{-19})(6 \times 10^{-4})}{2 \times 3.142 \times (9 \times 10^{-31})} \\ &= \frac{9.6}{18 \times 3.142} \times 10^8 = 16.97 \text{ MHz} \end{aligned}$$

Since the magnetic force does not change the kinetic energy of the charge,

$$\begin{aligned} \text{KE} &= \frac{1}{2} m_e v^2 = \frac{1}{2} (9 \times 10^{-31})(3 \times 10^7)^2 = \frac{81}{2} \times 10^{-17} \text{ J} \\ &= \frac{81}{2(1.6 \times 10^{-19})} \times 10^{-17} \text{ eV} \\ &= \frac{8.1}{3.2} \times 10^3 \\ &= 2.531 \text{ keV} \end{aligned}$$



Exercises | Q 5 | Page 249

An alpha particle accelerated to 10 MeV on entering a magnetic field of 1.88 T traces a circular path of radius 24.2 cm. Find the mass of the α -particle. [$e = 1.6 \times 10^{-19}$ C, $1 \text{ eV} = 1.6 \times 10^{-19}$ J]

SOLUTION

Data: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$E = 10 \text{ MeV} = 10^7 \times 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ J},$$

$$B = 1.88 \text{ T}, r = 0.242 \text{ m}, e = 1.6 \times 10^{-19} \text{ C}$$

Charge of an α -particle,

$$q = 2e = 2(1.6 \times 10^{-19}) = 3.2 \times 10^{-19} \text{ C}$$

$$r = \frac{(m_\alpha v)^2}{qB} \quad \text{and} \quad E = \frac{1}{2} m_\alpha v^2$$

$$\therefore r^2 = \frac{(m_\alpha v)^2}{q^2 B^2} \quad \text{and} \quad 2Em_\alpha = (m_\alpha v)^2$$

$$\therefore r^2 = \frac{2Em_\alpha}{q^2 B^2}$$

$$\therefore m_\alpha = \frac{(qBr)^2}{2E}$$

$$= \frac{[(3.2 \times 10^{-19})(1.88)(0.242)]^2}{2(1.6 \times 10^{-12})}$$

$$= (3.2 \times 10^{-26})(1.88 \times 0.242)^2$$

$$= (3.2 \times 10^{-26})(0.455)^2$$

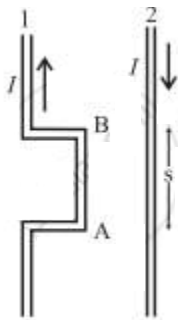
$$= (3.2 \times 10^{-26})(0.2070)$$

$$= 6.624 \times 10^{-27} \text{ kg}$$

Exercises | Q 6 | Page 249

Two wires shown in the figure are connected in a series circuit and the same current of 10 A passes through both, but in opposite directions. The separation between the two wires is 8 mm. The length AB is 22 cm. Obtain the direction and magnitude of the magnetic field due to current in wire 2 on the following figure segment AB of wire 1. Also, obtain the magnitude and direction of the force on that segment.

SOLUTION



Data: $I_1 = I_2 = 10 \text{ A}$, $s = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$, $l = 0.22 \text{ m}$

By right-hand grip rule, the direction of the magnetic field \vec{B}_2 due to the current in wire 2 at AB is into the page and its magnitude is

$$B_2 = \frac{\mu_0}{4\pi} \frac{2I}{s} = 10^{-7} \times \frac{2(10)}{8 \times 10^{-3}} = \frac{1}{4} \times 10^{-3} \text{ T}$$

The current in segment AB is upwards. Then, by Fleming's left-hand rule, the force on it due to \vec{B}_2 is to the left of the diagram, i.e., away from wire 1, or repulsive. The magnitude of the force is

$$\begin{aligned} F_{\text{on 1 by 2}} &= I_1 l B = (10)(0.22) \times \frac{1}{4} \times 10^{-3} \\ &= 5.5 \times 10^{-4} \text{ N} \end{aligned}$$

Exercises | Q 7 | Page 249

A very long straight wire carries a current of 5.2 A. What is the magnitude of the magnetic field at a distance 3.1 cm from the wire?

SOLUTION

Data: $I = 5.2 \text{ A}$, $a = 0.031 \text{ m}$, $\frac{\mu_0}{4\pi} = 10^{-7} \text{ T m/A}$

The magnetic induction,

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{4\pi} \frac{2I}{a} = 10^{-7} \times \frac{2(5.2)}{3.1 \times 10^{-2}}$$
$$= 3.35 \times 10^{-5} \text{ T}$$

Exercises | Q 8 | Page 249

Currents of equal magnitude pass through two long parallel wires having a separation of 1.35 cm. If the force per unit length on each of the wires is $4.76 \times 10^{-2} \text{ N}$, what is I ?

SOLUTION

Data: $I_1 = I_2 = I$, $s = 1.35 \times 10^{-2}$

$$F = \left(\frac{\mu_0}{4\pi} \right) \frac{2I_1 I_2 l}{s} = \left(\frac{\mu_0}{4\pi} \right) \frac{2I^2 l}{s}$$
$$\therefore I^2 = \frac{F}{I} \frac{s}{2(\mu_0/4\pi)}$$
$$= (4.76 \times 10^{-2}) \frac{1.35 \times 10^{-2}}{2 \times 10^{-7}} = 3.213 \times 10^3$$
$$\therefore I = \sqrt{32.13 \times 10^2} = 56.68 \text{ A}$$

Exercises | Q 9 | Page 249

The magnetic field at a distance of 2.4 cm from a long straight [current-carrying] wire is $16 \mu\text{T}$. What is the current through the wire?

SOLUTION

Data: $a = 2.4 \times 10^{-2} \text{ m}$, $B = 1.6 \times 10^{-5} \text{ T}$,

$$\frac{\mu_0}{4\pi} = 10^{-7} \text{ T m/A}$$

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0}{4\pi} \frac{2I}{a}$$



The current through the wire,

$$I = \frac{1}{\mu_0/4\pi} \frac{aB}{2} = \frac{1}{10^{-7}} = \frac{(2.4 \times 10^{-2})(1.6 \times 10^{-5})}{2} = 1.92 \text{ A}$$

Exercises | Q 10 | Page 249

The magnetic field at the centre of a circular current carrying loop of radius 12.3 cm is $6.4 \times 10^{-6} \text{ T}$. What is the magnetic moment of the loop?

SOLUTION

Data: $R = 12.3 \text{ cm} = 12.3 \times 10^{-2} \text{ m}$,

$B = 6.4 \times 10^{-6} \text{ T}$, $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$

$$\begin{aligned} B &= \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2R} \times \frac{2\pi R^2}{2\pi R^2} = \frac{\mu_0}{4\pi} \frac{2I(A)}{R^3} \quad (\because A = \pi R^2) \\ &= \frac{\mu_0}{4\pi} \frac{2\mu}{R^3} \end{aligned}$$

where $\mu = IA$ is the magnetic dipole moment of the coil.

$$\begin{aligned} \therefore \mu &= \frac{BR^3}{2(\mu_0/4\pi)} = \frac{(6.4 \times 10^{-6})(0.123)^3}{2 \times 10^{-7}} \\ &= 5.955 \times 10^{-2} \text{ J/T (or A m}^2\text{)} \end{aligned}$$

Exercises | Q 11 | Page 250

A circular loop of radius 9.7 cm carries a current 2.3 A. Obtain the magnitude of the magnetic field

(a) at the centre of the loop

(b) at a distance of 9.7 cm from the centre of the loop but on the axis.

SOLUTION

Data: $R = z = 9.7 \text{ cm} = 9.7 \times 10^{-2} \text{ m}$, $I = 2.3 \text{ A}$, $N = 1$

(a) At the centre of the coil:

The magnitude of the magnetic induction,

$$\begin{aligned} B &= \frac{\mu_0 NI}{2R} \\ &= \frac{(4\pi \times 10^{-7})(1)(2.3)}{2(9.7 \times 10^{-2})} = \frac{2 \times 3.142 \times 2.3}{9.7} \times 10^{-5} \\ &= 1.49 \times 10^{-5} \text{ T} \end{aligned}$$

(b) On the axis, at a distance $z = 0.2 \text{ m}$ from the coil:

$$\begin{aligned} B &= \frac{\mu_0}{4\pi} \frac{2\pi IR^2}{((R^2 + z^2))^{\frac{3}{2}}} \\ (R^2 + z^2)^{\frac{3}{2}} &= (2R^2)^{\frac{3}{2}} = 2\sqrt{2}R^3 \quad (\because R = z) \\ \therefore B &= \frac{\mu_0}{4\pi} \frac{2\pi IR^2}{2\sqrt{2}R^3} = \frac{\mu_0}{4\pi} \frac{\pi I}{\sqrt{2}R} \\ &= (10^{-7}) \frac{3.142 \times 2.3}{1.414 \times 9.7 \times 10^{-2}} \\ &= \frac{7.227}{13.72} \times 10^{-5} = 5.267 \times 10^{-6} \text{ T} = 5.267 \mu\text{T} \end{aligned}$$

Exercises | Q 12 | Page 250

A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field at the centre of the coil?

SOLUTION

Data: $N = 100$, $R = 8 \times 10^{-2} \text{ m}$, $I = 0.4 \text{ A}$, $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$

$$B = \frac{\mu_0 NI}{2R} = \frac{(4\pi \times 10^{-7})(100)(0.4)}{2(8 \times 10^{-2})} = 3.142 \times 10^{-4} \text{ T}$$

Exercises | Q 13 | Page 250

For proton acceleration, a cyclotron is used in which a magnetic field of 1.4 Wb/m^2 is applied. Find the time period for reversing the electric field between the two Ds.

SOLUTION

Data: $B = 1.4 \text{ Wb/m}^2$, $m = 1.67 \times 10^{-27} \text{ kg}$,

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$T = \frac{2\pi m}{qB}$$

$$t = \frac{T}{2} = \frac{\pi m}{qB} = \frac{3.142(1.67 \times 10^{-27})}{(1.6 \times 10^{-19})(1.4)}$$

$$= 2.342 \times 10^{-8} \text{ s}$$

Exercises | Q 14 | Page 250

A moving coil galvanometer has been fitted with a rectangular coil having 50 turns and dimensions $5 \text{ cm} \times 3 \text{ cm}$. The radial magnetic field in which the coil is suspended is of 0.05 Wb/m^2 . The torsional constant of the spring is $1.5 \times 10^{-9} \text{ Nm/degree}$. Obtain the current required to be passed through the galvanometer so as to produce a deflection of 30° .

SOLUTION

$N = 50$ turns, $C = 1.5 \times 10^{-9} \text{ Nm/degree}$, $A = lb = 5 \text{ cm} \times 3 \text{ cm} = 15 \text{ cm}^2 = 15 \times 10^{-4} \text{ m}^2$,
 $B = 0.05 \text{ Wb/m}^2$, $\theta = 30^\circ$

$$NIAB = C\theta$$



$$\therefore \text{The current through the coil, } I = \frac{C\theta}{NAB}$$

$$= \frac{1.5 \times 10^{-9} \times 30}{50 \times 15 \times 10^{-4} \times 0.05} = \frac{3 \times 10^{-5}}{5 \times 0.5} = 1.2 \times 10^{-5} \text{ A}$$

Exercises | Q 15 | Page 250

A solenoid of length L m and 5 cm in diameter has winding of 1000 turns and carries a current of 5 A. Calculate the magnetic field at its center along the axis.

SOLUTION

Data: $L = 3.142 \text{ m}$, $N = 1000$, $I = 5 \text{ A}$,

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$

The magnetic induction,

$$B = \mu_0 n I = \mu_0 \left(\frac{N}{L} \right) I$$

$$= (4\pi \times 10^{-7}) \left(\frac{1000}{3.142} \right) (5) = \frac{20 \times 3.142 \times 10^{-4}}{3.142}$$

$$= 2 \times 10^{-3} \text{ T}$$

Exercises | Q 16 | Page 250

A toroid of a central radius of 10 cm has windings of 1000 turns. For a magnetic field of $5 \times 10^{-2} \text{ T}$ along its central axis, what current is required to be passed through its windings?

SOLUTION

Data: Central radius, $r = 10 \text{ cm} = 0.1 \text{ m}$, $N = 1000$,

$$B = 5 \times 10^{-2} \text{ T}, \mu_0 = 4\pi \times 10^{-7} \text{ T·m/A}$$

The magnetic induction,

$$B = \frac{\mu_0 N I}{2\pi r} = \frac{\mu_0}{4\pi} \frac{2NI}{r}$$

$$\therefore 5 \times 10^{-2} = 10^{-7} \times \frac{2(1000)I}{0.1}$$

$$\therefore I = \frac{50}{2} = 25 \text{ A}$$

Exercises | Q 17 | Page 250

In a cyclotron, protons are to be accelerated. The radius of its D is 60 cm. and its oscillator frequency is 10 MHz. What will be the kinetic energy of the proton thus accelerated?

(Proton mass = $1.67 \times 10^{-27} \text{ kg}$, $e = 1.60 \times 10^{-19} \text{ C}$, $\text{eV} = 1.6 \times 10^{-19} \text{ J}$)

SOLUTION

Data: $R = 0.6 \text{ m}$, $f = 10^7 \text{ Hz}$, $m_p = 1.67 \times 10^{-27} \text{ kg}$,

$e = 1.6 \times 10^{-19} \text{ C}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

$$f = \frac{qB}{(2\pi m)_p} \text{ and } KE = \frac{q^2 B^2 R^2}{2m_p}$$

$$\therefore f^2 = \frac{q^2 B^2}{4\pi^2 m_p^2}$$

$$\therefore \frac{q^2 B^2}{2m_p} = 2\pi^2 f^2 m_p$$

$$\therefore KE = 2\pi^2 f^2 m_p R^2$$

$$= 2 \times 9.872 \times (10^7)^2 (1.67 \times 10^{-27}) (0.6)^2$$

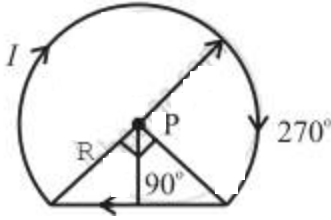
$$= 11.87 \times 10^{-13} \text{ J} = \frac{11.87 \times 10^{-13}}{1.6 \times 10^{-19}}$$

$$= 7.419 \times 10^6 \text{ eV}$$

$$= 7.419 \text{ MeV}$$

Exercises | Q 18 | Page 250

A wire loop of the form shown in the following figure carries a current I . Obtain the magnitude and direction of the magnetic field at P.



SOLUTION

The wire loop is in the form of a circular arc AB of radius R and a straight conductor BCA. The arc AB subtends an angle of $\Phi = 270^\circ = \frac{3\pi}{2}$ rad at the centre of the loop P. Since $PA = PB = R$ and C is the midpoint of AB, $AB = \sqrt{2}R$ and $AC = CB = \frac{\sqrt{2}R}{2} = \frac{R}{\sqrt{2}}$.

Therefore, $a = PC = \frac{R}{\sqrt{2}}$.

The magnetic inductions at P due to the arc AB and the straight conductor BCA are respectively,

$$B_1 = \frac{\mu_0}{4\pi} \frac{I\phi}{R} \text{ and } B_2 = \frac{\mu_0}{4\pi} \frac{2I}{a}$$

Therefore, the net magnetic induction at P is

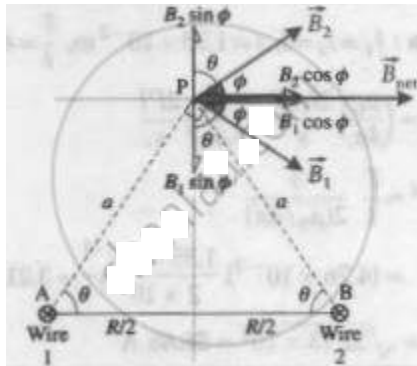
$$\begin{aligned} B_{\text{net}} &= B_1 + B_2 = \frac{\mu_0}{4\pi} \left[\frac{\phi}{R} + \frac{2}{R/\sqrt{2}} \right] \\ &= \frac{\mu_0}{4\pi} \frac{I}{R} [\phi + 2\sqrt{2}] = \frac{\mu_0}{4\pi} \frac{I}{R} \left[\frac{3\pi}{2} + 2\sqrt{2} \right] \end{aligned}$$

Exercises | Q 19 | Page 250

Two long parallel wires, both carrying current I directed into the plane of the page, are separated by a distance R. Show that at a point P equidistant from the wires and subtending an angle θ from the plane containing the wires, the magnitude of the

magnetic field is $B = \frac{\mu_0}{\pi} \frac{I}{R} \sin 2\theta$. What is the direction of the magnetic field?

SOLUTION



In the above figure, \vec{B}_1 and \vec{B}_2 are the magnetic fields in the plane of the page due to the currents in wires 1 and 2, respectively. Their directions are given by the right-hand grip rule: \vec{B}_1 is perpendicular to AP and makes an angle Φ with the horizontal. \vec{B}_2 is perpendicular to BP and also makes an angle Φ with the horizontal.

$$AP = BP = a = \frac{R/2}{\cos \theta}$$

$$\text{and } B_1 = B_2 = \frac{\mu_0}{4\pi} \frac{2I}{a} = \frac{\mu_0}{4\pi} \frac{2I(2 \cos \theta)}{R}$$

$$= \frac{\mu_0}{\pi} \frac{I}{R} \cos \theta$$

Since the vertical components cancel out, the magnitude of the net magnetic induction at P is

$$B_{\text{net}} = 2B_1 \cos \phi = 2B_1 \cos(90^\circ - \theta) = 2B_1 \sin \theta$$

$$= 2 \left(\frac{\mu_0}{\pi} \frac{I}{R} \cos \theta \right) \sin \theta = \frac{\mu_0}{\pi} \frac{I}{R} \sin 2\theta \text{ as required.}$$

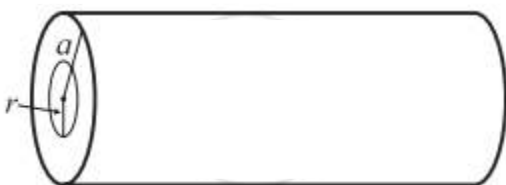
\vec{B}_{net} is in the plane parallel to that of the wires and to the right as shown in the figure.

Exercises | Q 20 | Page 250

The figure shows a cylindrical wire of diameter a , carrying a current I . The current density in the wire is in the direction of the axis and varies linearly with radial distance r

$$J_0 \frac{r}{a}.$$

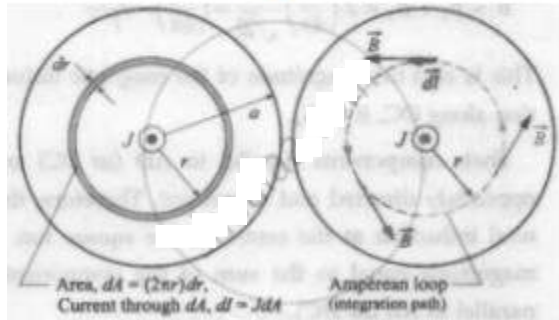
from the axis according to the relation $J =$ Obtain a magnetic field B inside the wire at a distance r from its center.



SOLUTION

Consider an annular differential element of radius r and width dr . The current through the area dA of this element is

$$dI = JdA = \left(J_0 \frac{r}{a} \right) 2\pi r dr = \frac{2\pi J_0 r^2 dr}{a} \quad \dots(1)$$



To apply the Ampere's circuital law to the circular path of integration, we note that the wire has perfect cylindrical symmetry with all the charges moving parallel to the wire. So, the magnetic field must be tangent to circles that are concentric with the wire. The enclosed current is the current within radius r . Thus,

$$\oint B dl = \mu_0 I_{\text{encl}}$$

$$\therefore \oint B dl = \mu_0 \int_0^r dl = \mu_0 \int_0^r \frac{2\pi J_0}{a} r^2 dr \quad \dots(2)$$

$$\therefore B(2\pi r) = \frac{\mu_0 2\pi J_0}{a} \left(\frac{r^3}{3} \right)$$

$$\therefore B = \frac{\mu_0 J_0}{3a} r^2 \quad \dots(3)$$

which is the required expression.

Exercises | Q 21 | Page 250

In the above problem, what will be the magnetic field B inside the wire at a distance r from its center, if the current density is uniform across the cross-section of the wire?

SOLUTION

The cross-section of a long straight wire of radius a that carries a current out of the page. Because the current is uniformly distributed over the cross-section of the wire, the magnetic field \vec{B} due to the current must be cylindrically symmetrical. Thus, along the Amperian loop of radius r ($r < a$), symmetry suggests that \vec{B} is tangent to the loop.

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl = B(2\pi r) \quad \dots(1)$$

Because the current is uniformly distributed, the current I_{encl} enclosed by the loop is proportional to the area encircled by the loop; that is,

$$I_{\text{encl}} = J\pi r^2$$

By right-hand rule, the sign of I_{encl} is positive. Then by Ampere's law,

$$B(2\pi r) = \mu_0 I_{\text{encl}} = \mu_0 J\pi r^2 \quad \dots(2)$$

$$\therefore B = \frac{\mu_0 J}{2} r \quad \dots(3)$$

OR

$$I_{\text{encl}} = I \frac{\pi r^2}{\pi a^2}$$

By right-hand rule, the sign of I_{encl} is positive. Then by Ampere's law,

$$\oint B dl = \mu_0 I_{\text{encl}}$$

$$\therefore B(2\pi r) = \mu_0 I \frac{r^2}{a^2} \quad \dots(4)$$

$$\therefore B = \frac{\mu_0 I}{2\pi a^2} r \quad \dots(5)$$